

§§ 7-8. Condition that two or more such shells should be replaceable by an equivalent single shell.

§§ 10-15. Empirical determination of constants for a single shell from observed refractions at Z.D. 45° and at the horizon.

§ 16. Extension of formulæ to two or more shells.

§ 17. Numerical results for one shell. The errors at 80° , 85° , and 89° are respectively $-6''$, $-40''$, $-281''$. The hypothesis cannot be said to fit the facts at all beyond Z.D. 60° or 70° at most. See Table I.

§§ 18-22. Numerical results for two shells. See Table I. The errors at 80° , 85° , and 89° are reduced to $-1''.5$, $-7''$, and $+2''$ —a very fair accordance for so obviously rough an assumption.

§§ 23-24. Numerical results for three shells: see Table V. The improvement is not great, much less marked than before; and it seems clear that the principal step was taken in passing from one shell to two.

§ 25. Some numerical values for heights, etc.

The Perturbations of Halley's Comet in the Past. Fourth Paper.
The period 760 to 1066. By P. H. Cowell, M.A., F.R.S., and
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We have again to acknowledge the kind assistance of Dr. Smart and Messrs. F. R. Cripps and Thos. Wright in these calculations.

In the last paper, *M.N.*, lxviii. 5, p. 378, we found March 27 for the approximate date of perihelion passage in 1066, and $44''.686$ for the value of n at that date. For the preceding passage we used the date given by Hind (989 September 12) for the purpose of computing the perturbations, and the following results indicate that this date is correct within a few days:—

Revolution 989-1066.

Planet.	Limits of u .	$\int dn$.	$\int d\pi$.	$\int d\zeta$.
Venus	0-30	+ '019	...	+ 532''
"	330-360	- '0035	...	...
Earth	0-30	+ '007	...	+ 196
"	330-360	- '013	...	...
Jupiter	0-90	+ '4535	- 531	+ 12807
"	90-270	- '2780	- 203	- 2905
"	270-360	- 1'2940	+ 708	+ 22
Saturn	0-90	- '0560	- 23	- 1578
"	90-270	+ '0754	- 6	+ 1979
"	270-360	- '2334	+ 7	+ 47
Uranus	0-360	+ '0030	...	+ 55
Sums		- 1'3200	- 48	+ 11155

For this and earlier revolutions we neglect the Neptune perturbations as trifling, and take those for Venus, Earth, and Uranus from curves constructed from the results already calculated; in the case of Venus and the Earth these curves are for the combined results of the mechanical quadratures and the definite integral. Needless to say, we do not claim that the results from these curves are absolutely accurate, but such accuracy is uncalled for, since our calculated Jupiter and Saturn results are liable to sensible errors through the uncertainty attaching to our assumed position of the comet's orbit plane, etc. The resulting value of n at 989 is $44''.686 + 1''.320 = 46''.006$. And calculated period in days = $\frac{1296000 - 11155}{46.006} = 27928$. Now 1066 March 27 = J.D. 2110500;

hence the calculated J.D. of previous passage is $2082572 = 989$ October 9. This is 27 days later than Hind's date, a quantity sufficiently small to confirm his identification. We can reduce the discordance slightly by altering our assumed dates of perihelion passage, since the observations are not precise enough to fix the exact day. If we take them as 989 September 15 and 1066 March 25, we obtain from the revolution 989-1066 the values of n , 989 $45''.969$, 1066 $44''.649$; from the following revolution we obtain 1066 $44''.688$, 1145 $44''.920$.

The mean value for 1066 is $44''.668$, to which corresponds $45''.988$ in 989.

Proceeding to the revolution 912-989 we first took Hind's date, 912 April 1, and we give the perturbations deduced from this assumption:—

Revolution 912-989.

Planet.	Limits of u .	$\int dn$.	$\int d\omega$.	$\int d\zeta$.
Venus	0-30°	+ '021	...	+ 588
"	330-360	- '016	...	...
Earth	0-30	- '0075	...	- 210
"	330-360	+ '0005	...	...
Jupiter	0-90	- '1905	+ 138	- 5503
"	90-270	+ '2064	- 73	+ 7936
"	270-360	+ '0966	- 598	+ 11
Saturn	0-90	+ '2251	- 19	+ 6402
"	90-270	- '0625	+ 15	+ 139
"	270-360	+ '0529	- 30	- 13
Uranus	0-360	+ '0102	...	+ 148
Sums		+ '3362	- 567	+ 9498

Taking $45''.988$ as the value of n in 989, that in 912 is $45''.652$, and calculated period in days = $\frac{1296000 - 9498}{45.652} = 28181$.

This gives J.D. 2054367 for the passage in 912 = 912 July 20.

This is nearly 4 months later than Hind's date, a larger quantity than is likely to arise from error in our calculation, and it is to be noticed that Hind's identification is antecedently very doubtful, being based only on observations on May 13, 15; we conclude that it is erroneous. There are vague references to other comets in 912, but nothing sufficiently precise to serve for identification. This is the first passage, reckoning back from the present time, that cannot be certainly identified with an observed comet.

Taking the calculated date 912 July 20, we obtain the following modified perturbations, 912 to 989. The value of $\int d\omega$ may be taken the same as before.

Revolution 912-989 revised.

Planet.	Limits of u .	$\int dn$.	$\int d\xi$.
Venus	0- 30°	+ '0086	+ 240"
"	330-360	- '016	...
Earth	0- 30	+ '003	+ 84
"	330-360	+ '0005	...
Jupiter	0- 90	- '2445	- 7026
"	90-270	+ '2284	+ 8576
"	270-360	+ '0966	+ 11
Saturn	0- 90	+ '2321	+ 6602
"	90-270	- '0655	+ 69
"	270-360	+ '0529	- 13
Uranus	0-360	+ '0102	+ 148
Sums		+ '306	+ 8691

This gives for the calculated period $\frac{1296000'' - 8691''}{45'' \cdot 682} = 28180$

days, practically the same as before.

Hence we take the date 912 July 19 as approximately correct for the perihelion passage, this result being from calculation alone, not from observation. As a check on its accuracy, we proceed to the revolution 837-912; a preliminary calculation indicates 837 February 28 for the preceding passage. It will be remembered that there is some question whether the numerous accounts of the brilliant object (or objects) that appeared in the spring of that year relate to one or to two comets. The date of perihelion passage of the earlier one, as investigated by Pingré, is indicated with tolerable precision as 837 March 1, the agreement with our calculated date being thus perfect. The other elements are also accordant except the position of the node, and it is well known that the indications of latitude in the Chinese accounts are somewhat vague, so that this is not a serious objection to the identification, especially as the comet was near the ecliptic throughout the

observations, so that a small error in the observed latitude would make a large error in the node. Hind took the second comet (if there were really two) as Halley's, finding 837 April 6 for the perihelion passage, but we think that the date March 1 is more likely to be correct.

Revolution 837-912.

Planet.	Limits of u .	$\int dn$.	$\int d\pi$	$\int d\xi$.
Venus	0-30	+ "020	..."	+ 550"
"	330-360	+ '0015	...	...
Earth	0-30	+ '023	...	+ 630
"	330-360	- '003	...	...
Jupiter	0-90	+ '975	- 84	+ 26892
"	90-270	- '226	- 493	- 3554
"	270-360	- 1'283	+ 707	+ 48
Saturn	0-90	- '038	+ 50	- 1040
"	90-270	+ '044	- 132	+ 633
"	270-360	- '018	+ 10	+ 8
Uranus	0-360	+ '0105	...	+ 258
Sums		- '494	+ 58	+ 24425

Taking n in 912 as $45''.682$ as found above, n in 837 = $46''.176$, and period in days = $\frac{1296000 - 24425}{46.176} = 27538$, which brings us to

J.D. 2026828 = 837 Feb. 25. We adopt this result as the date of perihelion passage in 837.

For the previous passage we have a fairly firm standpoint, for the comet of 760 was identified with Halley's by Laugier, from the orbit alone, before the intermediate passages had been determined, his date of perihelion passage being 760 June 11 = J.D. 1998810.

Revolution 760-837.

Planet.	Limits of u .	$\int dn$.	$\int d\pi$.	$\int d\xi$.
Venus	0-30	+ "001	..."	+ 27"
"	330-360	- '015	...	...
Earth	0-30	- '006	...	- 165
"	330-360	- '013	...	...
Jupiter	0-90	- '250	- 254	- 7164
"	90-270	+ '017	+ 191	+ 7484
"	270-360	+ '2755	- 274	- 45
Saturn	0-90	+ '071	- 90	+ 1995
"	90-270	- '011	+ 82	+ 1529
"	270-360	- '029	- 89	+ 1
Uranus	0-360	+ '0145	...	+ 360
Sums		+ '055	- 434	+ 4030

$$n \text{ 760} = 46''.176 - ''055 = 46''.121.$$

Period in days = $\frac{1296000 - 4030}{46.121} = 28013$, which brings us to

J.D. 1998815 = 760 June 15, which is 5 days later than Laugier's date.

This confirms the identity both of the comet of 760 and that of 837 with Halley's comet, and shows that we were right in taking the earlier date in 837 for the perihelion passage.

Now that the identification of the comet has been carried back to 760, it is of interest to quote the words of M. Laugier in the paper (*C.R.*, xv. p. 188) in which he identified the apparitions of 451 and 760 from the observations alone, and noticed that the average periodic time from 451 to 1378 was decidedly longer than in the subsequent seven returns:—"Le calcul des perturbations, s'il était faisable pendant cet énorme intervalle de temps, donnerait peut-être l'explication d'une variation aussi grande du temps périodique; mais, dans le cas où il ferait défaut, on pourrait se rappeler qu'une diminution analogue, quoique plus petite, a été observée dans les retours de la comète à courte période."

He goes on to speak of the retardation of Encke's comet, and of the suggestions that were put forward to account for this, viz. resisting medium, and Bessel's suggestion of the effect of loss of matter from the comet when passing perihelion, to which we may add the more recent theory of the effect of light-pressure on finely divided matter.

The calculation of the perturbations to which M. Laugier refers has now been carried out, not with absolute rigour, but sufficiently to reconcile theory and observation within a few days in each revolution; and we have found that the large changes in the periodic time are explicable on gravitational grounds alone. Our work does not justify us in asserting that the effect of the various non-gravitational causes referred to above is absolutely insensible; but it suffices to show that their combined effect does not amount to more than a week per revolution, and it is probably still less than this.